

# COMPETENCIES FOR SECONDARY MATHEMATICS TEACHERS, GRADES 7-12

In addition to the Arkansas Teaching Standards (ATS) and the competencies for the Teacher Excellence and Support System (TESS), including competencies regarding the knowledge and use of educational technology that reflect the International Society for Technology in Education standards, the teacher of Mathematics, grades 7-12, shall also demonstrate knowledge and competencies in the following areas:

## 1. KNOWING AND UNDERSTANDING MEANINGFUL MATHEMATICS

AR Algebra I, II & III, Calculus, Geometry, Statistics, and Quantitative Literacy Standards

NCTM/CAEP: Standard 1

Praxis 5161: Sections I & II

**NCTM/CAEP Standard 1:** *Demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications within and among mathematical domains of Number and Operations; Algebra and Functions; Statistics and Probability; Geometry, Trigonometry, and Measurement.*

*Arkansas teachers should exhibit knowledge and understanding of the essential concepts in each mathematical domain. Additional, specific competencies from Praxis content knowledge test specifications and Arkansas Standards are included. The intention of this is to augment and provide additional detail to the essential concepts.*

1.1 Essential Concepts in Number. Demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications of number including flexibly applying procedures, using real and rational numbers in contexts, developing solution strategies, and evaluating the correctness of conclusion. Major mathematical concepts in Number include number systems (particularly rational numbers); algorithmic and recursive thinking; number and set theory; ratio, rate of change, and proportional reasoning; and structure, relationships, operations, and representations

*Additional specific competencies in Number required for Arkansas teachers include:*

- Understand the structure of the natural, integer, rational, real, and complex number systems and how basic operations on numbers in these systems are performed
- Understand and apply the properties of exponents, including working with rational exponents and radicals
- Reason quantitatively and use understanding of units to solve problems (e.g., dimensional analysis, reasonableness of solutions)
- Understand how to solve problems involving ratios, proportions, averages, percents, and metric and traditional unit conversions
- Understand how to perform operations on matrices and apply matrices to solve problems, including in programming applications
- Represent and compare very large and very small numbers (e.g., scientific notation, orders of magnitude) and estimate and perform calculations on these numbers

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- Use number sense and proportional reasoning in real world settings to make and communicate decisions in quantitative analysis
- Understand how to use counting techniques such as the multiplication principle, permutations, and combinations
- Understand basic set theory (e.g., unions, differences, Venn diagrams)
- Understand the differences between discrete and continuous representations (e.g., data, functions) and how each can be used to model various phenomena

1.2 Essential Concepts in Algebra and Functions. Demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications of algebra and functions including how mathematics can be used systematically to represent patterns and relationships including proportional reasoning, to analyze change, and to model everyday events and problems of life and society. Essential Concepts in *Algebra and Functions* include algebra that connects mathematical structure to symbolic, graphical, and tabular descriptions; connecting algebra to functions; and developing families of functions as a fundamental concept of mathematics. Advanced concepts should include algebra from a more theoretical approach including relationship between structures (e.g., groups, rings, and fields) as well as formal structures for number systems and numerical and symbolic calculations.

Rewrite algebraic expressions in equivalent forms and choose the appropriate form of an algebraic expression for a given purpose

*Additional specific competencies in Algebra required for Arkansas teachers include:*

- Understand how to perform arithmetic operations on polynomials and rational expressions
- Understand and utilize the relationship between zeros of polynomial functions (including non-real complex zeros and graphical representations of real zeros) and factors of the related polynomial expressions
- Understand how to use polynomial identities (e.g., difference of squares, sum and difference of cubes) to solve problems
- Use equations and inequalities to describe relationships
- Justify the reasoning process used to solve equations, including analysis of potential extraneous solutions
- Use varied techniques (e.g., graphical, algebraic) to solve equations and inequalities in one variable
- Understand how varied techniques (e.g., graphical, algebraic, matrix) are used to solve systems of equations and inequalities
- Understand the concept of average rate of change over an interval for nonlinear functions and utilize it in problem-solving contexts

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- Recognize, extract, and interpret information about a linear equation presented in various forms (e.g., slope-intercept, point-slope, standard), including within a modeling context
- Understand sequences and define them recursively (e.g., arithmetic, geometric)

*Additional specific competencies in Functions required for Arkansas teachers include:*

- Understand the function concept and the use of function notation
- Understand how to find the domain and range of a function and a relation
- Analyze function behavior using different representations (e.g., graphs, mappings, tables, recursively-defined functions)
- Understand how to find and interpret the zero(s) of functions
- Understand how functions and relations are used to model relationships between quantities
- Understand how new functions are obtained from existing functions (e.g., compositions, transformations, inverses)
- Understand differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems
- Understand the relationship between points on the unit circle and the values of trigonometric functions for any given angle measure
- Understand how periodic phenomena are modeled using trigonometric functions
- Understand the derivation and application of trigonometric identities (e.g., Pythagorean, double angle, half angle, sum of angles, difference of angles)
- Understand how to interpret representations of functions of two variables (e.g., three-dimensional graphs, tables)
- Understand how to solve trigonometric, logarithmic, and exponential equations
- Apply knowledge of functions and equations in programming applications

1.3 Essential Concepts in Calculus. Demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications of calculus including the mathematical study of the calculation of instantaneous rates of change and the summation of infinitely many small factors to determine some whole. Essential Concepts in Calculus include limits, continuity, the Fundamental Theorem of Calculus, and the meaning and techniques of differentiation and integration.

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*Additional specific competencies in Calculus required for Arkansas teachers include:*

- Understand the meaning of a limit of a function and how to calculate limits of functions, conditions when the limit does not exist, and solve problems using the properties of limits
- Understand the derivative of a function as a limit, as the slope of a line tangent to a curve, and as a rate of change
- Understand what it means that a particular function is continuous at a given point
- Know the relationship between continuity and differentiability
- Understand how and when to use standard differentiation and integration techniques
- Understand how to analyze the behavior of a function (e.g., extrema, concavity, symmetry)
- Understanding how to apply derivatives to solve problems both theoretically and in a real-world context (e.g., related rates, optimization)
- Understand the foundational theorems of calculus (e.g., fundamental theorems of calculus, mean value theorem, intermediate value theorem)
- Understand integration as a limit of Riemann sums to compute area, volume, distance, or other accumulation processes
- Know how to determine the limits of sequences, if they exist

1.4 Essential Concepts in Statistics and Probability. Demonstrate and apply understandings of statistical thinking and the major concepts, procedures, knowledge, and applications of statistics and probability, including how statistical problem solving and decision making depend on understanding, explaining, and quantifying the variability in a set of data to make decisions and understanding the role of randomization and chance in determining the probability of events. Essential Concepts in Statistics and Probability include quantitative literacy, visualizing and summarizing data, statistical inference, probability, and applied problems

*Additional specific competencies in Statistics and Probability required for Arkansas teachers include:*

- Understand how to summarize, represent, and interpret data collected from measurements on a single variable (e.g., box plots, dot plots, normal distributions)
- Understand how to summarize, represent, and interpret data collected from measurements on two variables, either categorical or quantitative (e.g., scatterplots, time series)
- Understand how to create and interpret linear regression models (e.g., rate of change, intercepts, correlation coefficient)

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- Understand how to make inferences and justify conclusions from samples, experiments, and observational studies
- Understand the concept of independence and understand how to compute probabilities of simple events, probabilities of compound events, and conditional probabilities
- Know how to make informed decisions using probabilities and expected values
- Understand how to find probabilities involving finite sample spaces and independent trials, including the use of the counting techniques (e.g., fundamental counting principles, permutations, combinations)
- Understand normal distributions
- Using statistical and probabilistic reasoning to draw conclusions, to make decisions, and to evaluate outcomes of decision

1.5 Essential Concepts in Geometry, Trigonometry, and Measurement. Demonstrate and apply understandings of major mathematics concepts, procedures, knowledge, and applications of geometry, including using visual representations for numerical functions and relations, data and statistics, and networks, to provide a lens for solving problems in the physical world. Essential Concepts in *Geometry, Trigonometry, and Measurement* include transformations, geometric arguments, reasoning and proof, applied problems, and non-Euclidean geometries

*Additional specific competencies in Geometry required for Arkansas teachers include:*

- Know the properties of lines (e.g., parallel, perpendicular, intersecting) and angles
- Know and apply properties of triangles, quadrilaterals (e.g., parallelogram, rectangle, rhombus) and other polygons
- Understand and investigate transformations in the plane and apply their properties
- Understand congruence and similarity, including congruence and similarity theorems and use of transformations to define congruence and similarity
- Know how to prove geometric theorems, such as those about lines and angles, triangles, and parallelograms
- Understand how geometric constructions are made with a variety of tools and methods
- Understanding how trigonometry is applied to triangles, including the definition of trigonometric functions in right triangles
- Understand and apply theorems about circles
- Find arc length and area measurements of sectors of circles

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## 2. KNOWING AND USING MATHEMATICAL PROCESSES

NCTM/CAEP: Standard 2

NCTM-MP

- Know how to translate between a geometric description (e.g., focus, asymptotes, directrix) and an equation for a conic section
- Understand how to represent geometric objects in coordinate geometry to algebraically prove simple geometric theorems
- Use perimeter, area, surface area, and volume formulas to solve problems
- Know how to visualize relationships (e.g., cross-section, nets, rotations) between two-dimensional and three-dimensional objects
- Apply geometric concepts in modeling and real-world situations

*NCTM/CAEP Standard 2: Demonstrate, within or across mathematical domains, their knowledge of and ability to apply the mathematical processes of problem solving; reason and communicate mathematically; and engage in mathematical modeling. Apply technology appropriately within these mathematical processes.*

2.1 Demonstrate a range of mathematical problem-solving strategies to make sense of and solve nonroutine problems (both contextual and non-contextual) across mathematical domains

2.2 Organize mathematical reasoning and use the language of mathematics to express mathematical reasoning precisely, both orally and in writing, to multiple audiences

2.3 Understand the difference between the mathematical modeling process and models in mathematics; engage in the mathematical modeling process and demonstrate ability to model mathematics

*NCTM-MP: The Standards for Mathematical Practices describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education.*

- Make sense of problems and persevere in solving them
- Reason abstractly and quantitatively
- Construct viable arguments and critique the reasoning of others
- Model with mathematics
- Use appropriate tools strategically
- Attend to precision
- Look for and make use of structure
- Look for and express regularity in repeated reasoning

2.4 Understand the importance of providing students with opportunities to learn mathematics that enable them to think analytically and creatively for everyday problem-solving and preparation for the workforce, college, citizenship, and life

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## 3. KNOWING STUDENTS AND PLANNING FOR MATHEMATICAL LEARNING

NCTM/CAEP: Standard 3

*NCTM/CAEP Standard 3: Apply knowledge of students and mathematics to plan rigorous and engaging mathematics instruction supporting students' access and learning, and develop mathematics instruction that provides equitable, culturally responsive opportunities for all students to learn and apply mathematics concepts, skills, and practices.*

3.1 Identify and use students' individual and group differences to plan rigorous and engaging mathematics instruction that supports students' meaningful participation and learning

3.2 Identify and use students' mathematical strengths to plan rigorous and engaging mathematics instruction that supports students' meaningful participation and learning

3.3 Understand that teachers' interactions impact individual students by influencing and reinforcing students' mathematical identities, positive or negative, and plan experiences and instruction to develop and foster positive mathematical identities

## 4. TEACHING MEANINGFUL MATHEMATICS

NCTM/CAEP: Standard 4

NCTM-PA

*NCTM/CAEP Standard 4: Implement effective and equitable teaching practices to support rigorous mathematical learning for a full range of students. Establish rigorous mathematics learning goals, engage students in high cognitive demand learning, use mathematics specific tools and representations, elicit and use student responses, develop conceptual understanding and procedural fluency, and pose purposeful questions to facilitate student discourse.*

4.1 Establish rigorous mathematics learning goals for students based on mathematics standards and practices

4.2 Select or develop and implement high cognitive demand tasks to engage students in mathematical learning experiences that promote reasoning and sense making

4.3 Select mathematics-specific tools, including technology, to support students' learning, understanding, and application of mathematics and to integrate tools into instruction

4.4 Select and use mathematical representations to engage students in examining understandings of mathematics concepts and the connections to other representations

4.5 Use multiple student responses, potential challenges, and misconceptions, and they highlight students' thinking as a central aspect of mathematics teaching and learning

4.6 Use conceptual understanding to build procedural fluency for students through instruction that includes explicit connections between concepts and procedures

4.7 Pose purposeful questions to facilitate discourse among students that ensures that each student learns rigorous mathematics and builds a shared understanding of mathematical ideas



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## 5. ASSESSING IMPACT ON STUDENT LEARNING

NCTM/CAEP: Standard 5

*NCTM/CAEP Standard 5: Assess and use evidence of students' learning of rigorous mathematics to improve instruction and subsequent student learning. Analyze learning gains from formal and informal assessments for individual students, the class as a whole, and subgroups of students disaggregated by demographic categories, and they use this information to inform planning and teaching.*

5.1 Select, modify, or create both informal and formal assessments to elicit information on students' progress toward rigorous mathematics learning goals

5.2 Collect information on students' progress and use data from informal and formal assessments to analyze progress of individual students, the class as a whole, and subgroups of students disaggregated by demographic categories toward rigorous mathematics learning goals

5.3 Use the evidence of student learning of individual students, the class as a whole, and subgroups of students disaggregated by demographic categories to analyze the effectiveness of their instruction with respect to these groups. Propose adjustments to instruction to improve student learning for each and every student based on the analysis

## 6. SOCIAL AND PROFESSIONAL CONTEXT OF MATHEMATICS TEACHING AND LEARNING

NCTM/CAEP: Standard 6

*NCTM/CAEP Standard 6: Aspire to become reflective mathematics educators who collaborate with colleagues and other stakeholders to grow professionally, to support student learning, and to create more equitable mathematics learning environments.*

6.1 Seek to create more equitable learning environments by identifying beliefs about teaching and learning mathematics, and associated classroom practices that produce equitable or inequitable mathematical learning for students

6.2 Reflect on their impact on students' mathematical identities and develop professional learning goals that promote students' positive mathematical identities

6.3 Communicate with families to share and discuss strategies for ensuring the mathematical success of their children

6.4 Collaborate with colleagues to grow professionally and support student learning of mathematics

## 7. SECONDARY FIELD EXPERIENCES AND CLINICAL PRACTICE

NCTM/CAEP: Standard 7

*NCTM/CAEP Standard 7: Effective teachers of secondary mathematics engage in a planned sequence of field experiences and clinical practice in diverse settings under the supervision of experienced and highly qualified mathematics teachers. They develop a broad experiential base of knowledge, skills, effective approaches to mathematics teaching and learning, and professional behaviors across both middle and secondary settings that involve a diverse range and varied groupings of students. Candidates experience a full-time student teaching/internship in secondary mathematics supervised by university or college faculty with secondary mathematics teaching experience or equivalent knowledge.*

7.1 Participate in a diverse range of field experiences and clinical practice in both middle grade and secondary settings with highly qualified mathematics teachers